

# Program of the Conference

Thursday May, 30	Friday May, 31	Saturday June, 1	Sunday June, 2	Monday June, 3
7 :30 <i>Breakfast</i>	7 :30 <i>Breakfast</i>	7 :30 <i>Breakfast</i>	7 :30 <i>Breakfast</i>	7 :30 <i>Breakfast</i>
9 :15-10 :30 <b>P. Dèbes</b> <i>Pre-Galois theory</i>	9 :15-10 :30 <b>L. Bary-Soroker</b> <i>Number Theory in Function Fields</i>	9 :15-10 :30 <b>B. Collas</b> <i>Galois Arithmetics of Perverse Sheaves in Genus one</i>	9 :15-10 :30 <b>D. Harbater</b> <i>Local-global principles for torsors under tori</i>	9 :30 <i>Leaving Tatihou</i>
10 :30-11 :00 <i>Coffee break</i>	10 :30-11 :00 <i>Coffee break</i>	10 :30-11 :00 <i>Coffee break</i>	10 :30-11 :00 <i>Coffee break</i>	
11 :00-12 :00 <b>F. Pop</b> <i>On the birational Grothendieck-Teichmueller Group</i>	11 :00-12 :00 <b>A. Smeets</b> <i>Brauer-Manin failure for a simply connected fourfold, via orbifold Mordell</i>	11 :00-12 :00 <b>S. Checcoli</b> <i>Fields of algebraic numbers with bounded local degrees and their Galois groups.</i>	11 :00-12 :00 <b>V. Mehmeti</b> <i>Berkovich Curves and the Local-Global Principle</i>	
12 :30-15 :30 <i>Lunch</i>	12 :30-13 :30 <i>Lunch</i> <i>Free afternoon or defenses afternoon</i>	12 :30-15 :30 <i>Lunch</i>	12 :30-15 :30 <i>Lunch</i>	
<i>Free time</i>	13 :30-15 :30 <b>F. Legrand</b> Soutenance d'Habilitation à Diriger les Recherches <i>Quelques contributions à l'arithmétique des extensions de <math>k(T)</math> et de leurs spécialisations</i>	<i>Free time</i>	<i>Free time</i>	
15 :30-16 :30 <b>D. Harari</b> <i>Artin-Mazur-Milne duality theorem revisited</i>	15 :30-16 :00 <i>Coffee break</i>	15 :30-16 :30 <b>D. Neftin</b> <i>Grouping extensions into families</i>	15 :30-16 :30 <b>A. Fehm</b> <i>The minimal ramification problem for function fields</i>	
16 :30-17 :00 <i>Coffee break</i>	16 :00-18 :00 <b>F. Motte</b> Soutenance de Thèse <i>De la géométrie à l'arithmétique en théorie inverse de Galois</i>	16 :30-17 :00 <i>Coffee break</i>	16 :30-17 :00 <i>Coffee break</i>	
17 :00-18 :00 <b>C. Demarche</b> <i>Local-global principles for homogeneous spaces over global fields</i>		17 :00-18 :00 <b>Y. Harpaz</b> <i>Supersolvable descent and Galois theory</i>	17 :00-18 :00 <b>F. Legrand</b> <i>Finite groups as automorphism groups of field extensions</i>	
18 :00-19 :00 <b>R. Cluckers</b> <i>Rational points of bounded heights</i>	18 :30 Buffet des soutenances <i>(Coktail Dinner)</i>	18 :00-19 :00 <b>O. Wittenberg</b> <i>Massey products in the Galois cohomology of number fields</i>	18 :00-19 :00 <b>B. Deschamps</b> <i>Inverse Galois theory over skew fields</i>	
19 :30 <i>Dinner</i>		19 :30 <i>Conference Banquet</i>	19 :30 <i>Dinner</i>	

# Abstracts

**Lior Bary-Soroker** — *Number Theory in Function Fields.*

I will describe some of the recent threads in the study of number theory in function fields, some of the techniques that is used, some of the applications of the theory, and some of the challenges.

**Sara Checcoli** — *Fields of algebraic numbers with bounded local degrees and their Galois groups.*

It is known that, if  $K$  is a number field and  $L/K$  is an infinite Galois extension, then the local degrees of  $L$  are uniformly bounded at all rational primes if and only if the group  $\text{Gal}(L/K)$  has finite exponent. Also motivated by some problems concerning the Bogomolov property (a height gap property for sets of algebraic numbers), one can ask whether the simple non-uniform boundedness of the local degrees of  $L$  is still equivalent to some (weaker) group theoretical property of  $\text{Gal}(L/K)$ . We will show that this is not the case in general, by exhibiting several groups that admit two different realisations over a given number field, one with bounded local degrees at a given set of primes and one with infinite local degrees at the same primes.

**Raf Cluckers** — *Rational points of bounded heights.*

I will present an overview of recent progress on bounds for the number of rational points of bounded height on various objects which are sometimes algebraic, sometimes analytic. A common ingredient is the determinant method, initiated by Bombieri - Pila in this context, and varied upon by e.g. Heath-Brown and Salberger. I will explain this method and its variants, and relate to a question by Serre (sometimes called the dimension growth conjecture) and to some non-archimedean counting problems. This concerns various joint works, with Castryck - Dittmann - Nguyen, Comte - Loeser, Forey - Loeser, Pila - Wilkie (all on arxiv).

**Benjamin Collas** — *Galois Arithmetics of Perverse Sheaves in Genus one.*

In the context of genus zero moduli spaces of curves, explicit computations of perverse sheaves leads to arithmetic properties of two Galois natures : the inverse Galois realization of symplectic and orthogonal groups (Dettweiler-Reiter & Völklein), and some Tannakian motivic results with the computation of periods relations (Deligne-Terasoma) or the realization of  $G_2$  as motivic Galois group (Dettweiler-Reiter). The goal of this talk is to present similar results in genus one.

Having briefly introduced how (perverse) l-adic realizations give a favorable theoretical and computational context for formulating these two Galois approaches, I will first present how genus one allows in Regular Inverse Galois theory to go beyond the Fried-Völklein branch-cycle rational rigidity barrier, then how the pentagon and hexagon monoidal conditions suffice to identifies  $G_2$  as a Tannakian group of motivic nature. The former relies on Katz' convolution algorithm, the latter on local systems of Katz-Beauville type and both on explicit monodromy computations.

This is joint work with M. Dettweiler, S. Reiter and W. Sawin.

**Pierre Dèbes** — *Pre-Galois Theory.*

We introduce the notion of pre-Galois extensions : they are those field extensions  $E/k$  which become Galois after some linearly disjoint Galois base change  $L/k$ . When  $k = \kappa(T)$  for some field  $\kappa$  and  $L = \bar{\kappa}(T)$ , they are called geometrically Galois. We develop a pre-Galois theory and investigate the corresponding variant of the inverse Galois problem. We provide answers in situations where the classical analogs are unknown. On the other hand, we also show that if the pre-Galois variant holds for all finite groups over a field  $k$ , then the classical one holds as well over  $k$ . (Joint work with D. Harbater)

**Cyril Demarche** — *Local-global principles for homogeneous spaces over global fields of positive characteristic.*

Using Artin-Mazur duality for fppf cohomology, we study Galois cohomology of (complexes of) tori over a global field of positive characteristic. In particular, we get a Poitou-Tate exact sequence relating the global Galois cohomology groups to the local ones. We will explain how to apply those results to study rational points on homogeneous spaces of linear algebraic groups over such fields. This is a joint work with David Harari.

**Bruno Deschamps** — *Inverse Galois theory over skew fields.*

There exists a Galois theory for skew fields, generalizing the commutative case and very close of it in his overall results. As an invitation to a stroll in this still poorly explored forest of skew fields, we will explain in this talk how we can investigate the inverse Galois problem over  $K(t)$  where  $K$  is a skew field and  $K(t)$  denote the field of skew rational fractions over  $K$  with central indeterminate. The most immediate and significant application of this investigation will be that every finite group occurs as a Galois group over  $\mathbb{H}(t)$ . This is joint work with François Legrand.

**Arno Fehm** — *The minimal ramification problem for function fields.*

For a finite group  $G$ , the minimal ramification problem asks for the minimal number of ramified primes in Galois extension of the field of rational numbers with Galois group  $G$ . Boston-Markin gave a conjectural answer for any group  $G$ , but only some special cases are proven. I will discuss the minimal ramification problem for rational function fields over finite and other fields, with a focus on symmetric groups. Joint work with Lior Bary-Soroker and Alexei Entin.

**David Harari** — *Artin-Mazur-Milne duality theorem revisited.*

Fppf cohomology is a natural generalization of Galois and étale cohomology. In this talk we will explain how a duality theorem (due to Artin-Mazur-Milne, but whose proof in the literature up to now seems incomplete) for finite group schemes can be proved in this context. We will also give various new finiteness statements, including examples and counterexamples. This is joint work with Cyril Demarche.

**David Harbater** — *Local-global principles for torsors under tori.*

This talk concerns local-global principles for torsors over semi-global fields, i.e. function fields of curves over a complete discrete valuation ring  $R$ . Previous work of the speaker and collaborators have focused on the case of torsors under rational linear algebraic groups. The focus of this talk will be the case of torsors under tori defined over  $R$  that are not assumed rational. The geometry of the curve comes into play in computing the obstructions to such local-global principles.

**Yonathan Harpaz** — *Supersolvable descent and Galois theory.*

It is conjectured by Colliot-Thélène that if  $X$  is a smooth, proper and rationally connected variety over a number field  $k$  then the set of rational points is dense in the Brauer set of  $X$ . Suppose that such a variety  $X$  is equipped with the action of a finite group  $G$ . If we know Colliot-Thélène's conjecture for every twist of  $X$  by a class in  $H^1(k, G)$ , can we deduce the same for (a smooth and proper model of)  $X/G$ ? A positive answer to this question will imply the inverse Galois problem for  $G$ . In a joint work with Olivier Wittenberg we show that the answer to this question is positive when  $G$  is supersolvable, that is, admits a filtration by normal subgroups with cyclic consecutive quotients. This has applications to the problem of finding  $G$ -Galois extensions from which a given element is a norm, generalizing similar results recently obtained by Frei, Loughran and Newton in the case where  $G$  is abelian.

**François Legrand** — *Finite groups as automorphism groups of field extensions.*

In this talk, I shall present several results on the realizability of finite groups as automorphism groups of field extensions (not necessarily Galois). In particular, I shall explain how to obtain an analogue of the Dèbes-Deschamps conjecture in this context. This is based on joint works with Bruno Deschamps, Arno Fehm and Elad Paran.

**Vlerë Mehmeti** — *Berkovich Curves and the Local-Global Principle.*

Patching was first introduced as an approach to the Inverse Galois Problem. The technique was then extended to a more algebraic setting and used to prove a local-global principle by D. Harbater, J. Hartmann and D. Krashen. I will present an adaptation of the method of patching to the setting of Berkovich analytic curves. This will then be used to prove a local-global principle for function fields of curves with respect to two families of overfields: one appearing quite naturally in Berkovich's theory, and the other (which consists of completions) evoking a resemblance with more classical versions of this principle. The results that will be presented generalize those of the above-mentioned authors.

**Danny Neftin** — *Grouping extensions into families.*

A standard approach to statistically analyze a set of extensions is by grouping it into a family, that is, an extension whose coefficients are parameters, and each extension in  $S$  is obtained by specializing those parameters. The smaller the number of parameter is, the more accurate the statistics are. We shall discuss the problem of parametrizing all or part of the extensions with Galois group  $G$  using one or two parameter families, a local approach to this problem, and its connection to two conjectures of Colliot-Thelene.

**Florian Pop** — *On the birational Grothendieck-Teichmüller Group.*

The Grothendieck-Teichmüller group  $GT$  arose from Grothendieck's "dessins d'enfants." It is still a widely open question whether the canonical embeddings of the absolute Galois group of the rationals into  $GT$  (e.g. using Deligne's "tangent points") are isomorphisms, or even whether the two groups are isomorphic as abstract profinite groups. In this talk I will define birational variants of  $GT$ , and answer in positive the above questions for the birational variants of  $GT$  under discussion. The technical tools originate from techniques developed to tackle Bogomolov's birational anabelian program.

**Arne Smeets** — *Brauer-Manin failure for a simply connected fourfold, via orbifold Mordell.*

Almost one decade ago, Poonen constructed the first examples of algebraic varieties over global fields for which Skorobogatov's étale Brauer-Manin obstruction does not explain the failure of the Hasse principle. By now, several constructions are known, but they all share common geometric features such as large fundamental groups. I will explain how to construct simply connected fourfolds over global fields of positive characteristic for which the Brauer-Manin machinery fails. Contrary to earlier work in this direction, this construction does not rely on major conjectures. The main technical tool is a Mordell-type theorem for Campana's "geometric orbifolds" over function fields of positive characteristic. (Joint work with Stefan Kebekus and Jorge Pereira.)

**Olivier Wittenberg** — *Massey products in the Galois cohomology of number fields.*

(Joint work with Yonatan Harpaz.) Let  $k$  be a field and  $p$  be a prime. According to a conjecture of Mináč and Tân, Massey products of  $n > 2$  classes in  $H^1(k, \mathbb{Z}/p\mathbb{Z})$  should vanish whenever they are defined. We establish this conjecture when  $k$  is a number field, for any  $n$ . This constraint on the absolute Galois group of  $k$  was previously known to hold when  $n = 3$  and when  $n = 4, p = 2$ .